# Frequency space, fourier transforms, and image analysis

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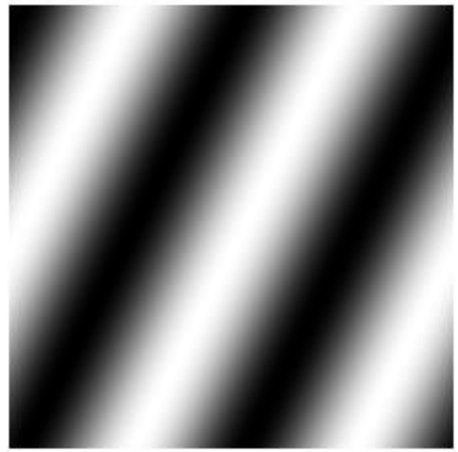
# Think of Images as Sums of Waves

… or "spatial frequency components"

one wave

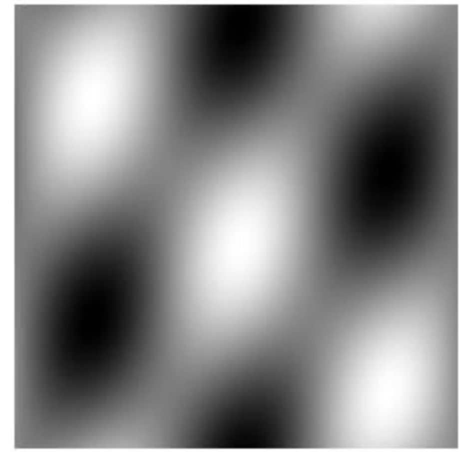
+

another wave



=

(2 waves)



+ (…) =

(25 waves)

+ (…) =

(10000 waves)

# Frequency Space

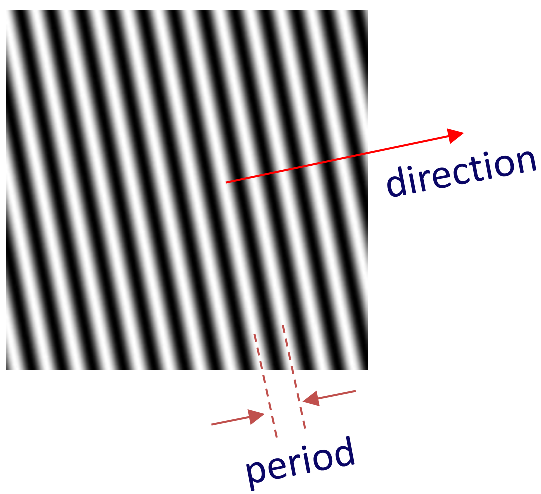
To ***describe*** a wave,
we need to specify its:

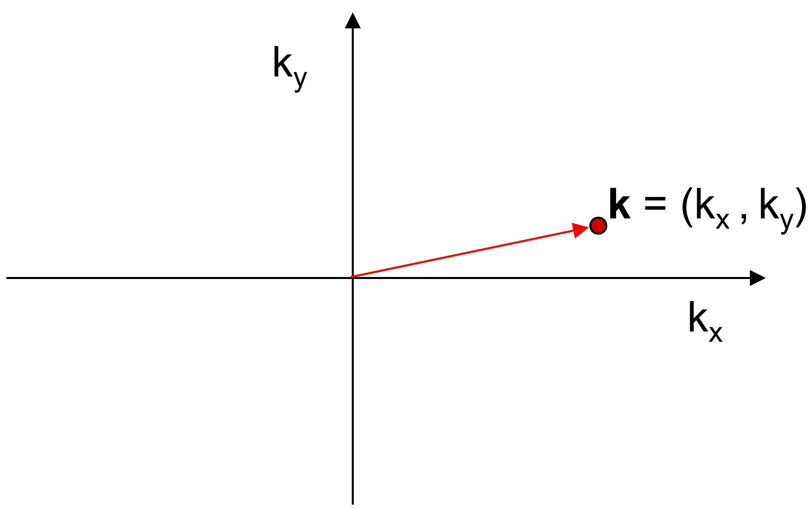
- Frequency (how many periods/meter?)
- Direction
- Amplitude (how strong is it?)
- Phase (where are the peaks & troughs?)

Can describe it by
a *value* at a *point*

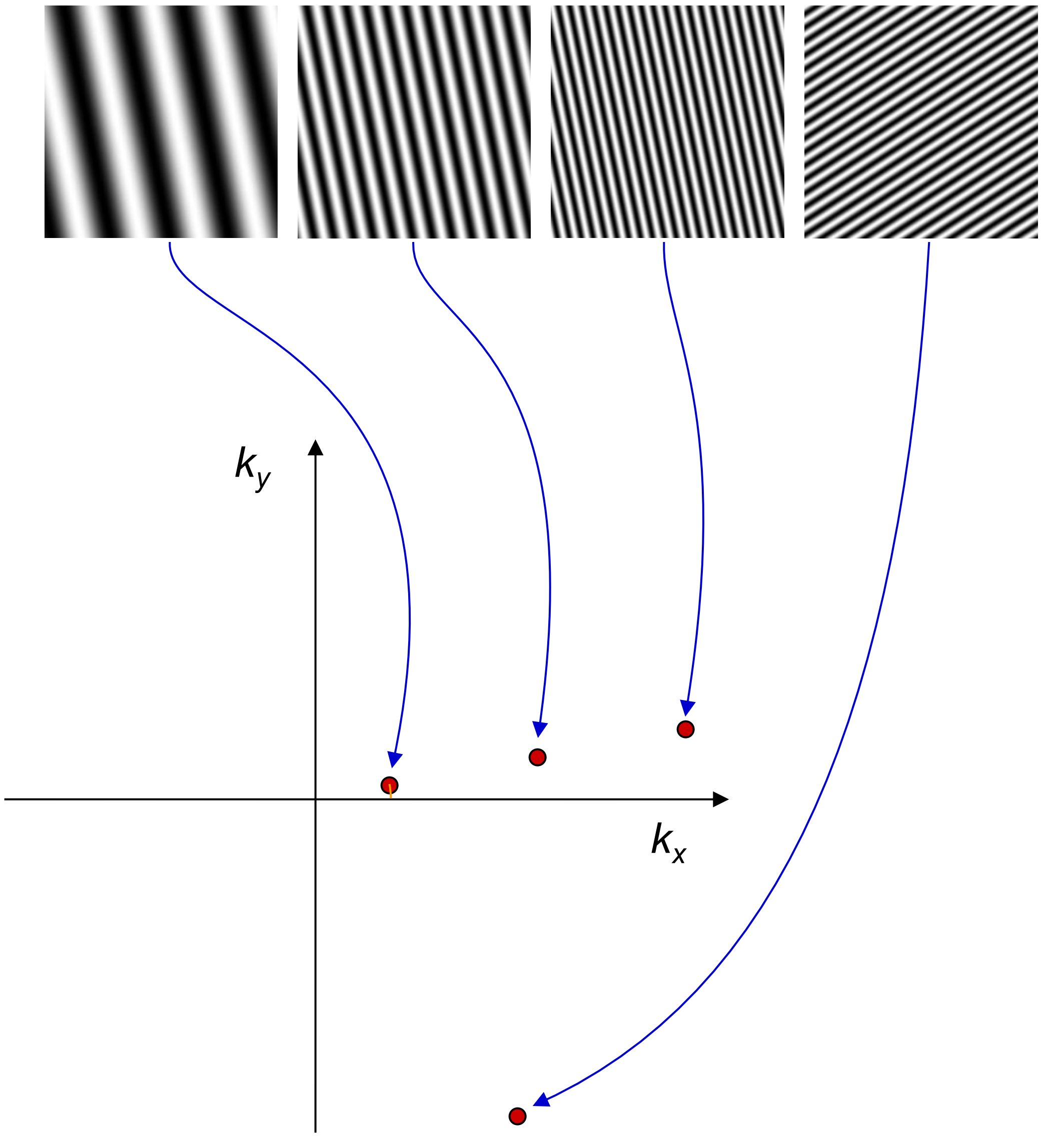
- Distance from origin
- Direction from origin
- Magnitude of value
- Phase of value

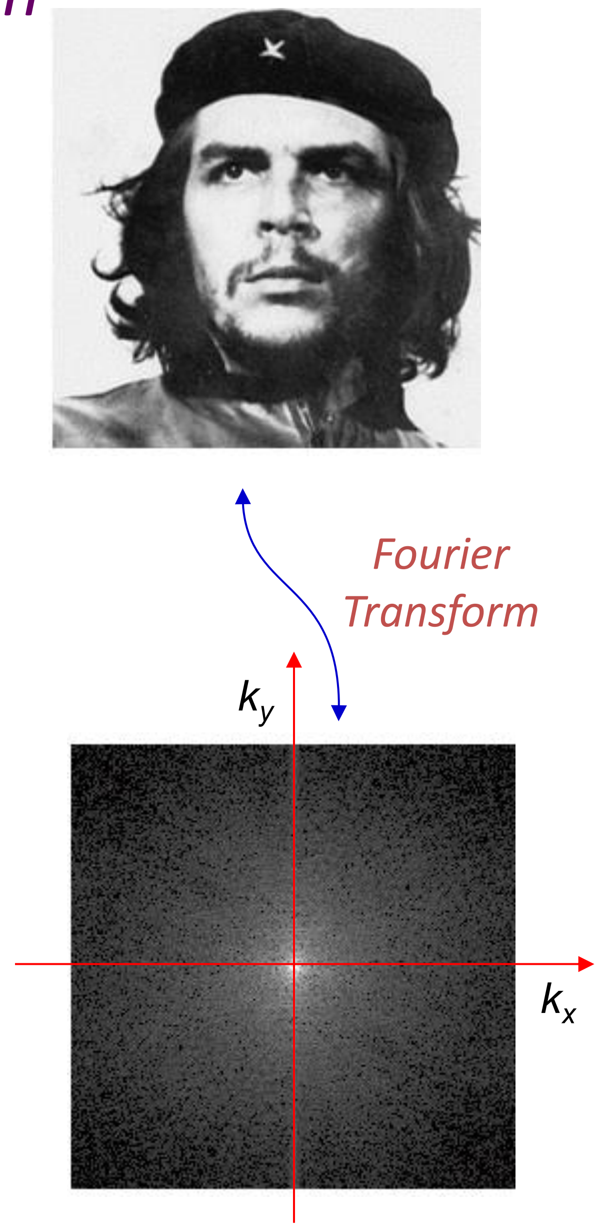
complex



# Frequency Space
# and the *Fourier Transform*



# Properties of the Fourier Transform

$$F(\mathbf{k}) = \int f(\mathbf{r}) e^{2\pi i \mathbf{k} \bullet \mathbf{r}} d\mathbf{r}$$

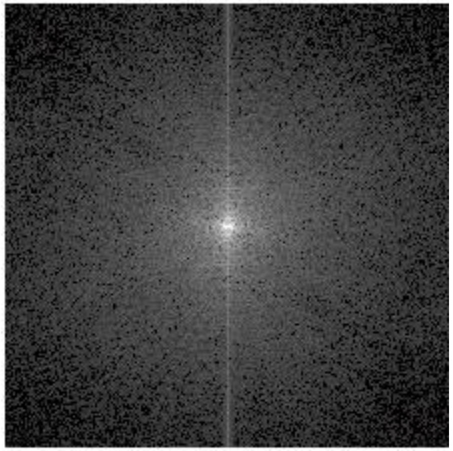
Completeness:

The Fourier Transform contains **all** the information of the original image

Symmetry:

The Fourier Transform of the Fourier Transform is the original image

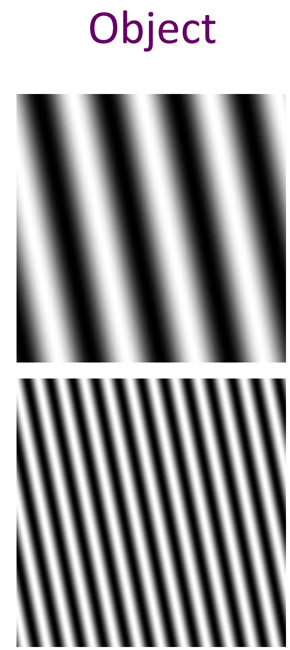
*Fourier transform*

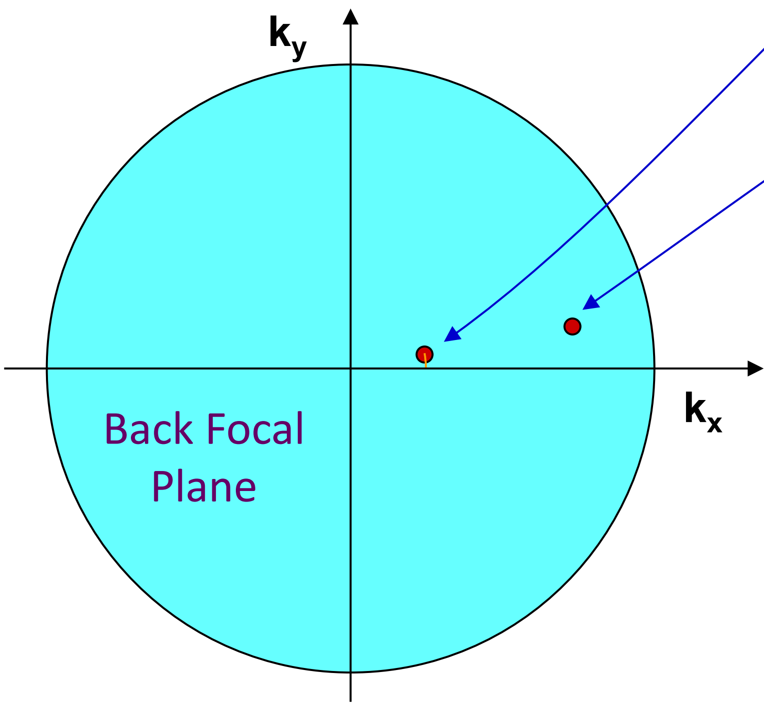


# Frequency space and imaging

Imagine a sample composed of a single frequency sine wave

It gives rise to a diffraction pattern at a single angle, which maps to a single spot in the back focal plane

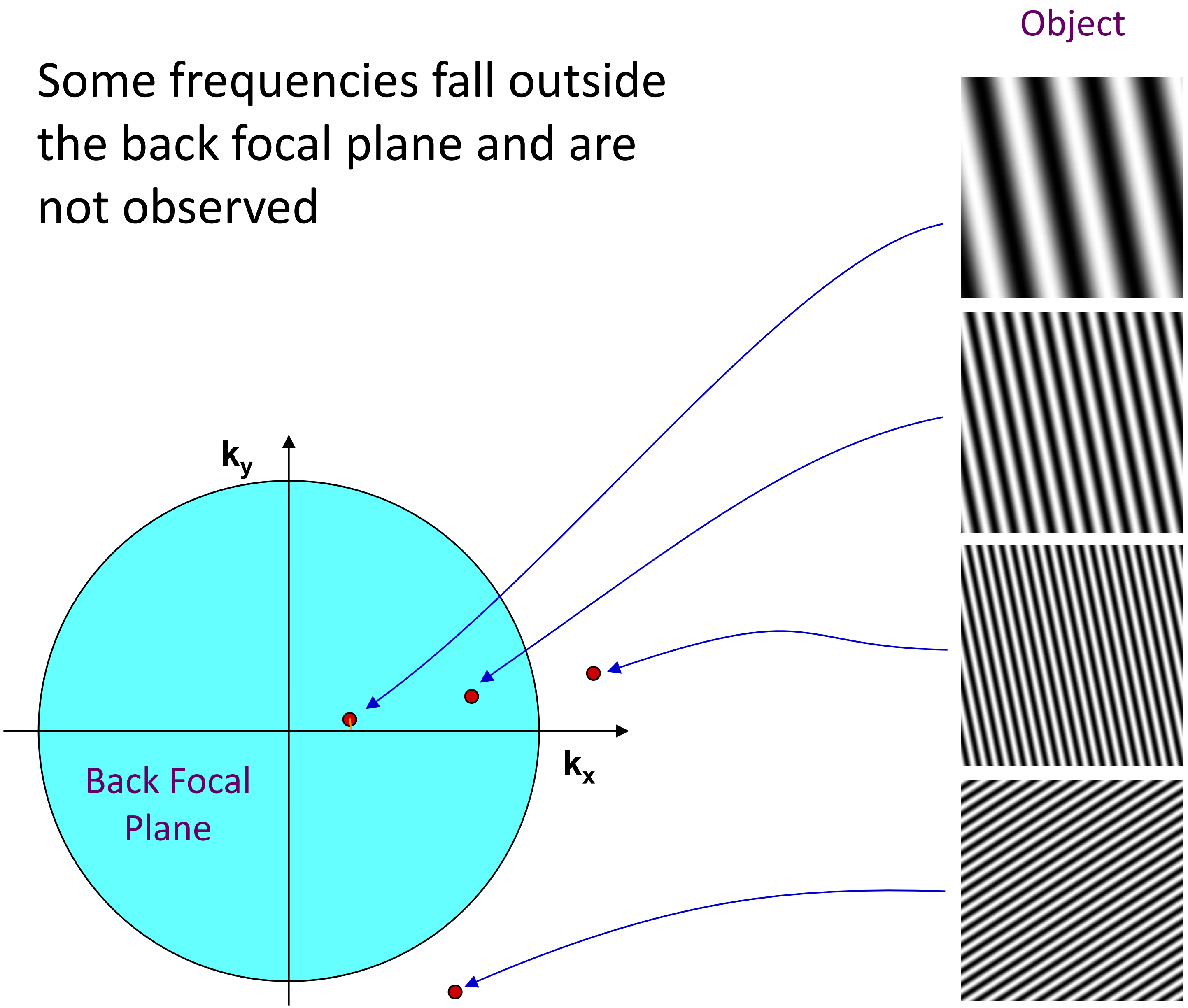


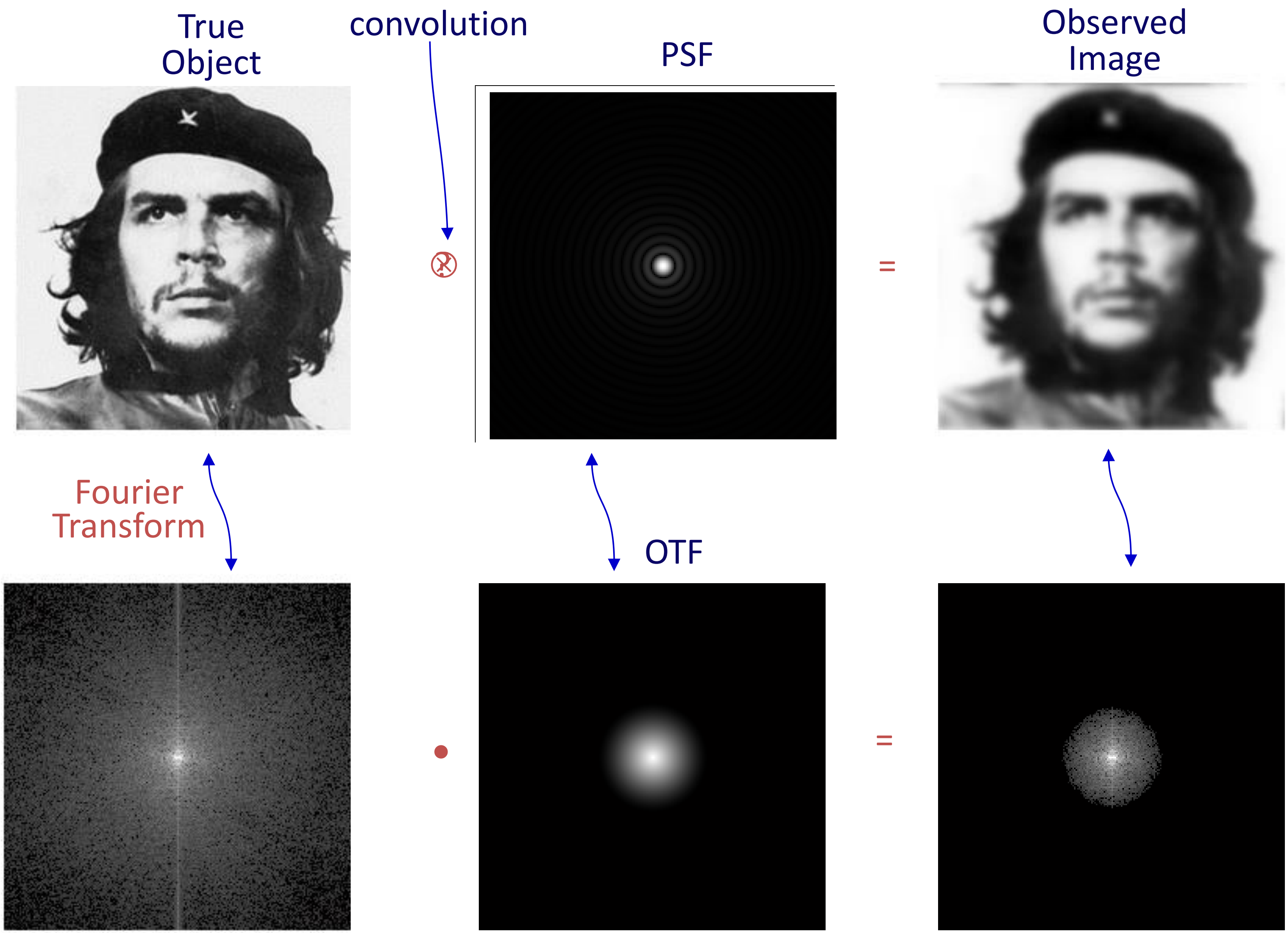

The back focal plane IS the Fourier transform of your sample!

# Frequency space and resolution

Some frequencies fall outside the back focal plane and are not observed



# The OTF and Imaging



# Convolutions

$$(f \otimes g)(r) = \int f(a)\, g(r-a)\, da$$

Why do we care?

- They are everywhere…
- The **convolution theorem**:

If $h(\mathbf{r}) = (f \otimes g)(\mathbf{r})$,

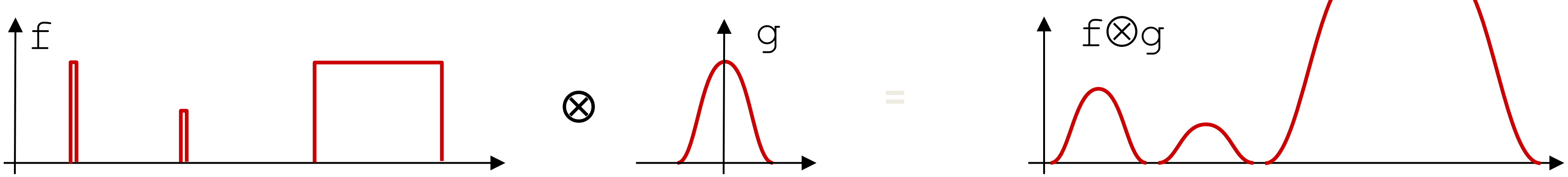
then $\tilde{h}(\mathbf{k}) = \tilde{f}(k)\ \tilde{g}(\mathbf{k})$

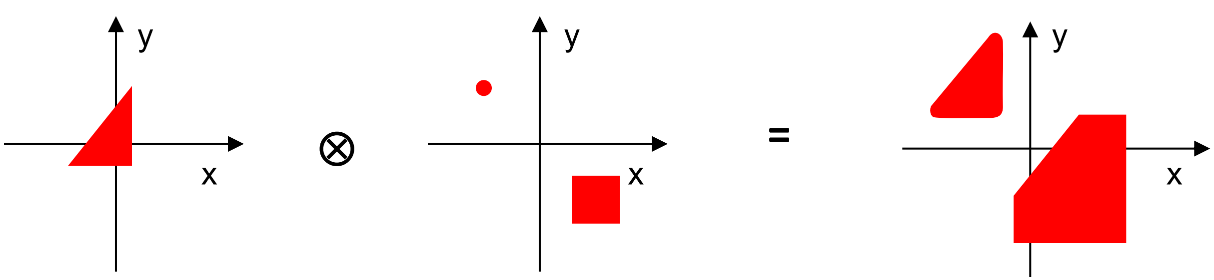
A convolution in real space becomes
a product in frequency space & vice versa

Symmetry: $g \otimes f = f \otimes g$

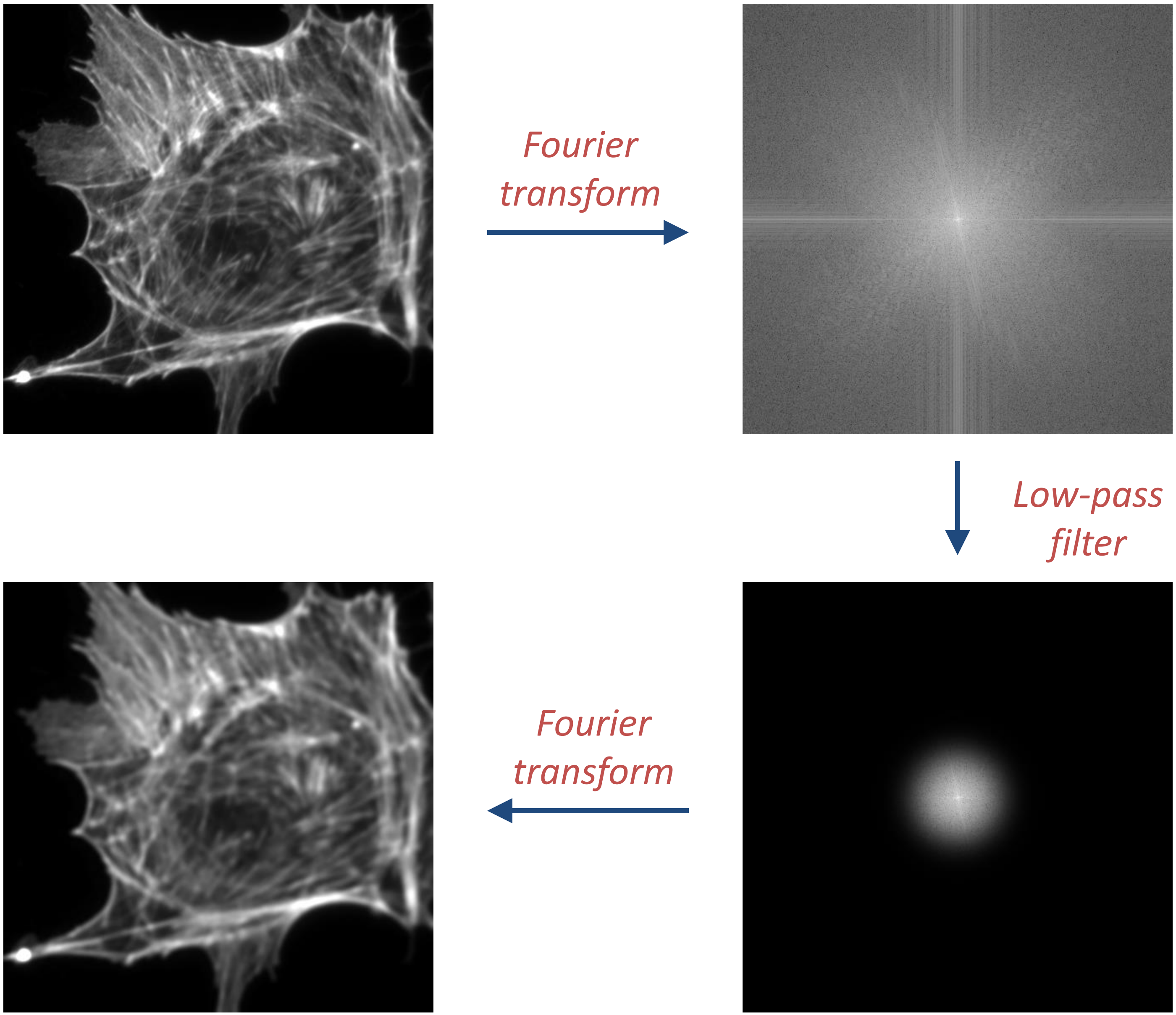
So what **is** a convolution, intuitively?

- "Blurring"
- "Drag and stamp"



# Filtering with Fourier transforms



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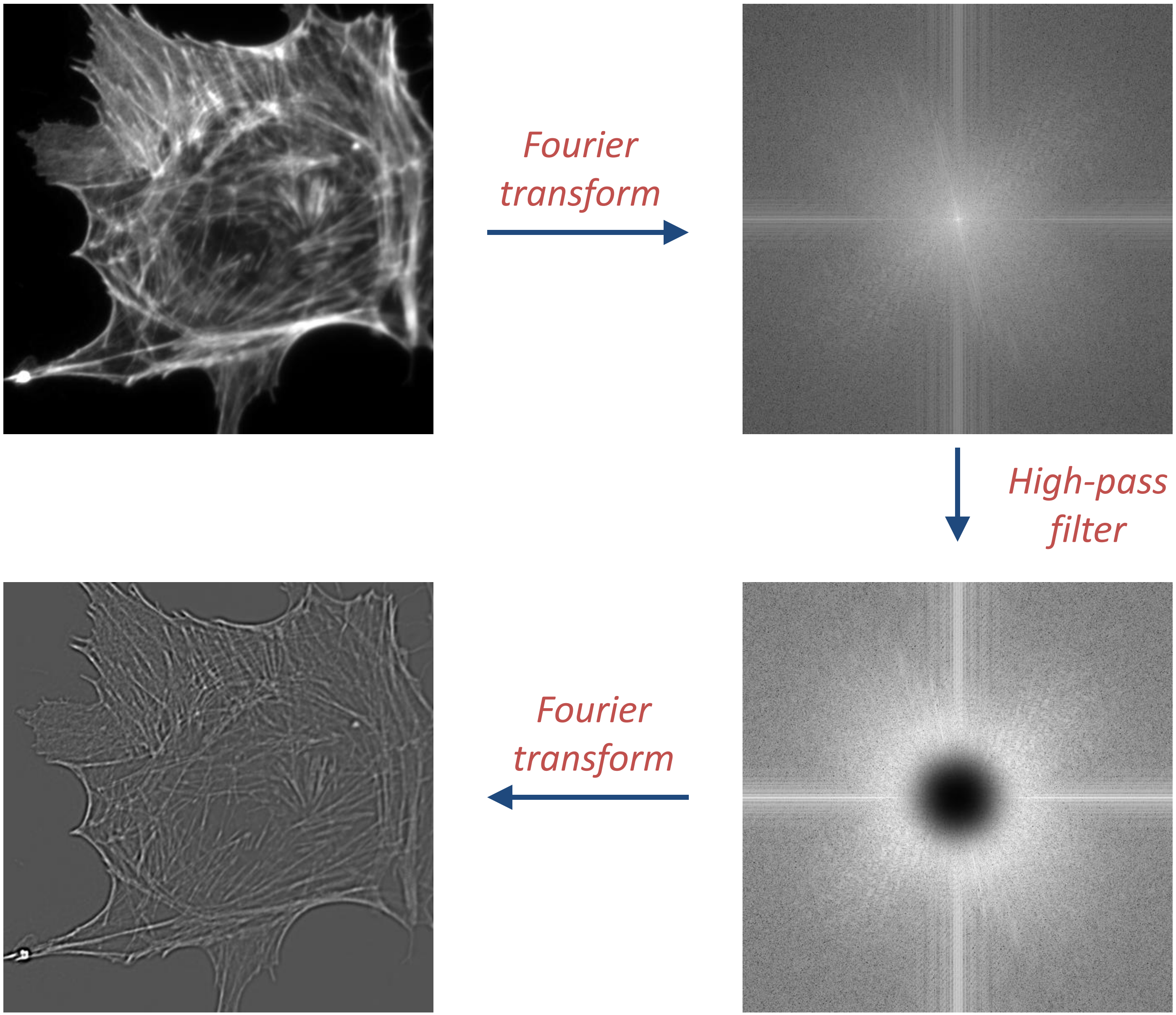
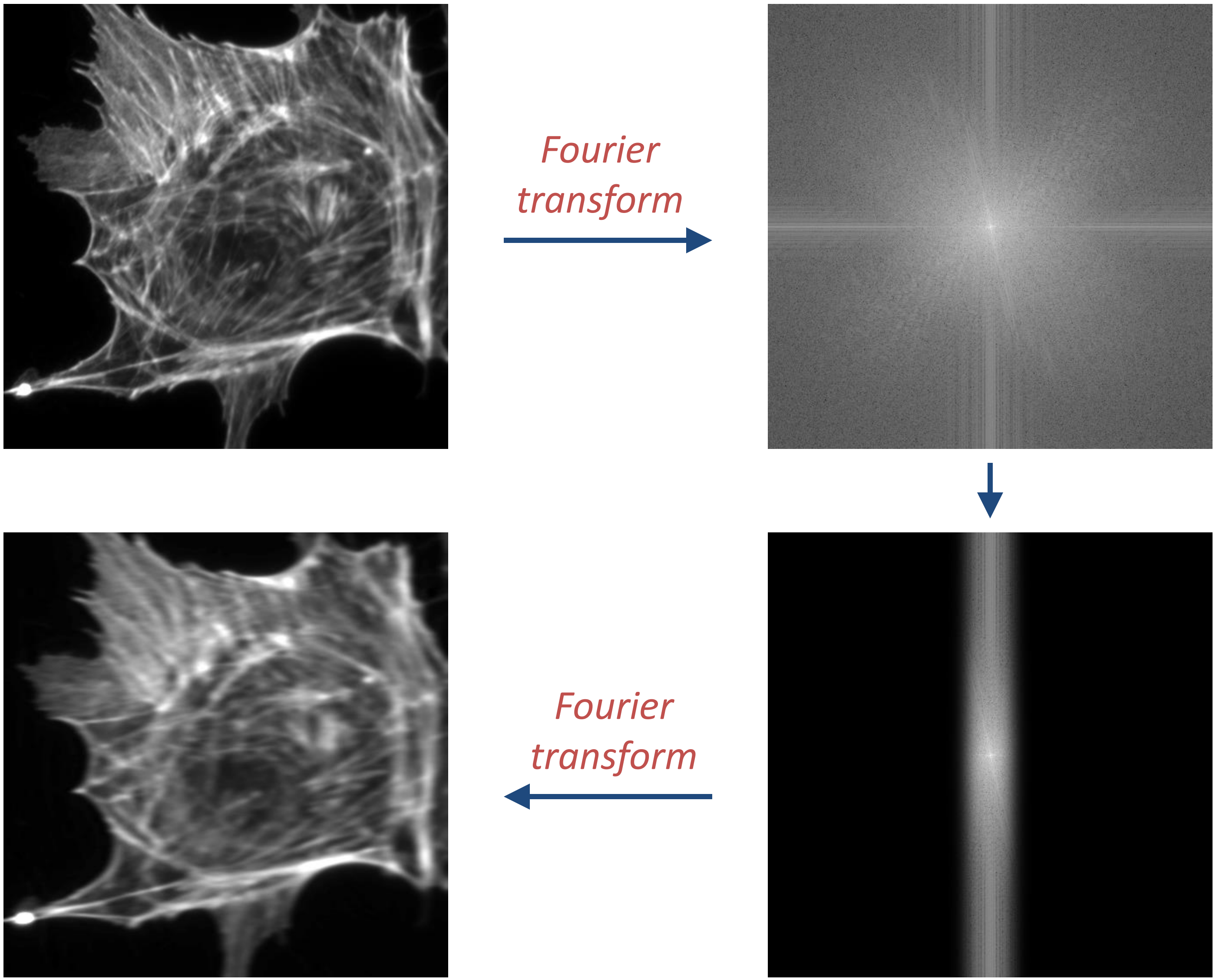
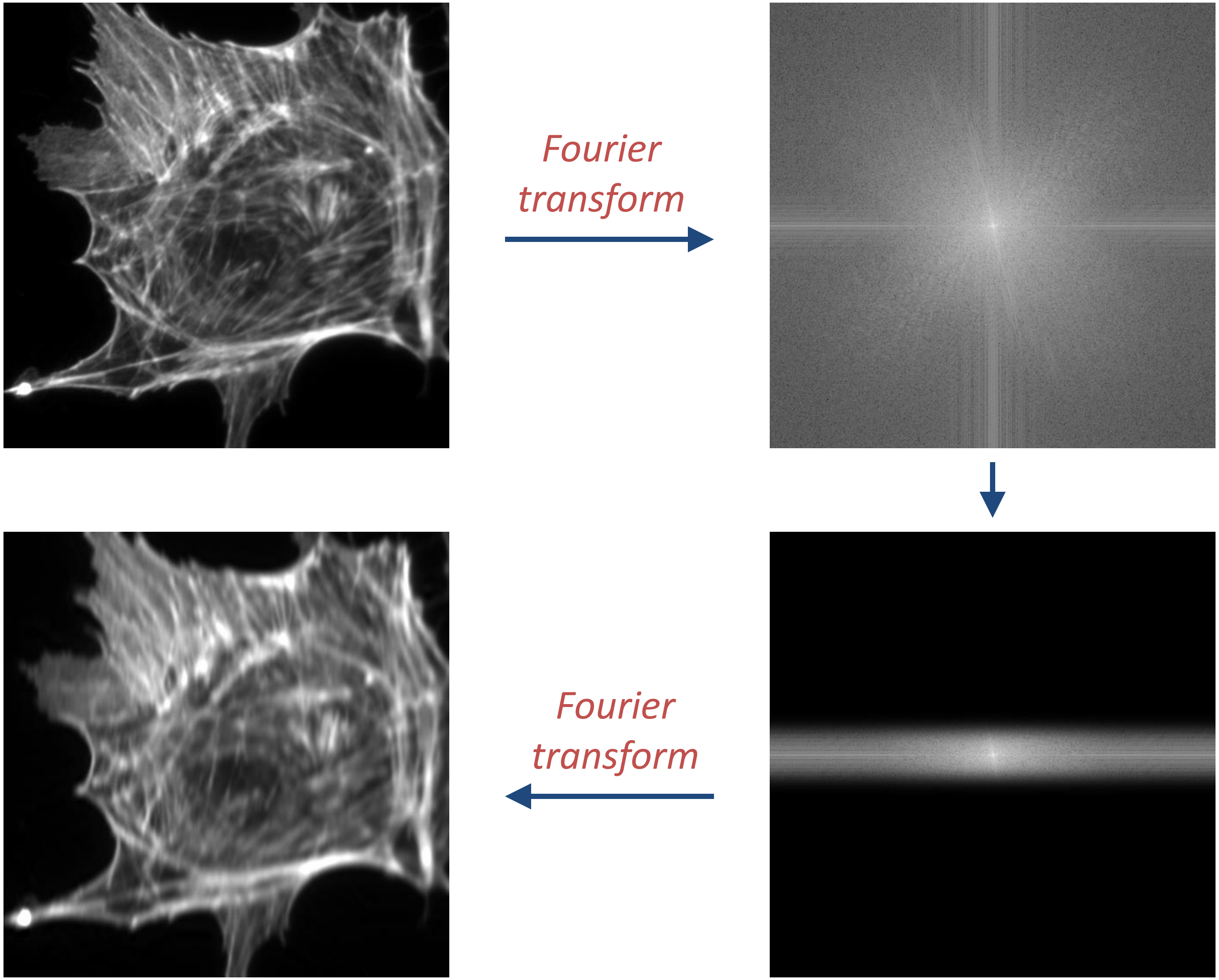


# Stranger filters




Blurs vertical objects

# Stranger filters




Blurs horizontal objects

# Acknowledgements

- Mats Gustafsson